# Improvement of Bit Error Rate (BER) of Adaptive Orthogonal Frequency Division Multiplexing (AOFDM) Systems under Rayleigh Fading Condition by using Convolutional Coding

#### Md. Tanvir Hasan

**Abstract**— In a fourth-generation wireless system, cellular providers have the opportunity to offer data access to a wide variety of devices. The cellular network would become a data network on which cellular phones could operate — as well as any other data device. For efficient transmission and reception in fourth-generation wireless system, adaptive orthogonal frequency division multiplexing system or AOFDM is considered as a potential approach. In AOFDM, adaptive transmission scheme is engaged in accordance with the channel fading condition with OFDM to develop the system performance. This paper demonstrates a technique to improve the BER of AOFDM by using convolutional coding. The BER performance of OFDM using un-coded adaptive modulation using quadrature amplitude modulation (QAM) and phase shift keying (PSK) have also been investigated to depict its comparison with the proposed coded system. The obtained results show that a significant improvements in terms of bit error rate (BER) can be achieved using the proposed system.

**Index Terms**— Adaptive modulation, bit error rate (BER), channel coding, MPSK, MQAM, orthogonal frequency division multiplexing (OFDM), transmission blocking.

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### **1** INTRODUCTION

THE main rewards of OFDM are its tolerance against L multipath delay spread and spectral efficiency due to its overlapping capability in the frequency domain. The computational efficiencies of inverse Fast Fourier Transformation (IFFT) and Fast Fourier Transformation (FFT) operations as the modulation and demodulation scheme of OFDM have make it further attractive to deploy. OFDM is a special form of multi-carrier transmission technique in which a single high rate data stream is divided into multiple low rate data streams. These data streams are then modulated using subcarriers which are orthogonal to each other. In this way the symbol rate on each sub channel is greatly reduced, and hence the effect of inter-symbol interference (ISI) due to channel dispersion in time caused by multipath delay spread is reduced.

However, in OFDM, each subcarrier is attenuated individually under the frequency-selective and fast fading channel. The channel performance may be highly fluctuating across the

• Md. Tanvir Hasan

subcarriers and varies from symbol to symbol [1]. If the same

fixed transmission scheme is employed for all OFDM subcarriers, the error probability is dominated by the OFDM subcarriers with highest attenuation resulting in a poor performance. Therefore, in case of frequency selective fading the error probability decreases very slowly with increasing average signal-to-noise ratio (SNR) [2].

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This problem can be moderated if individual OFDM subcarriers are modulated using different modulation schemes. Adaptive OFDM schemes have to be adapted to the SNR of the individual subcarriers. This will considerably improve the BER performance of an OFDM system. However, most of the AOFDM systems do not consider the channel coding across the bits due to higher efficiency of OFDM transmission scheme. But this assumption not always stays in favorable limit specially, in a highly fading environment that can be modeled by Rayleigh distribution. This paper describes an efficient way to improve the BER performance of an AOFDM system by introducing convolutional coding in the system. Efficiency of the resultant system is verified via simulation and analysis.

Many adaptive transmission techniques have been presented in the literature. The combination of adaptive modulation with OFDM was proposed as early as 1989 by Kalet which was further developed by Chow [3] and Czylwik [2]. Specifically the results obtained by Czylwik showed that the

Master of Engineering in Telecommunications

<sup>•</sup> Bachelor of Science in Electrical and Electronic Engineering

<sup>•</sup> American International University – Bangladesh (AIUB)

E-mail:wirelesstanvir@gmail.com

required SNR for the BER target 10<sup>-3</sup> can be reduced by 5dB to 15dB compared to fixed OFDM depending on the scenario of radio propagation. The performance of turbo-coded adaptive modulation is investigated in [4]. Three different modulation mode allocation algorithms were discussed and compared. Further studies on the application of turbo code in adaptive modulation and coding is conducted in [5]. This paper proposed an optimal approach based on prediction of the average BER over all subcarriers. In [6], an adaptive OFDM system with changeable pilot spacing has been proposed. The results showed that a significant improvement in the BER performance is achieved with sacrificing a small value of the total throughput of the system. A work is done on several strategies on bit and power allocation for multi-antenna assisted OFDM systems in [7]. They found out that sometimes power and bit adaptation is required for efficient exploitation of wireless channels in some system conditions.

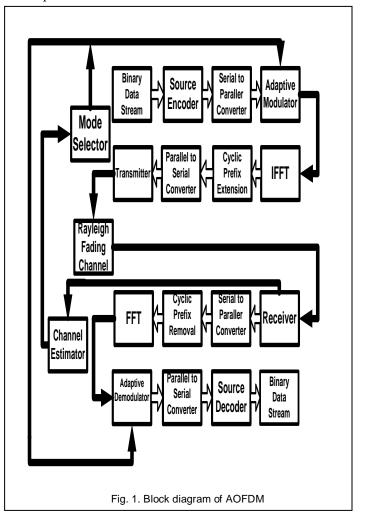
The performance analysis of OFDM systems with adaptive subcarrier bandwidth is investigated by [8]. Further investigations on subcarrier adaptive modulation scheme of Subcarriers are grouped into sub-band the average instantaneous SNR for each sub-band are calculated The average instantaneous SNR are compared with the thresholds value to choose the mode The information about modulation and coding used are sent to the transmitter using a feedback channel. precoded OFDM is presented in [9] under multipath channels.

This paper is organized as follows: The adaptive procedure and system model of AOFDM is described in Section 2. Section 3 describes an introductory overview of Rayleigh fading condition. A preliminary demonstration of convolutional coding is provided in Section 4. The proposed AOFDM system is illustrated in Section 5. Section 6 examines the contribution of the proposed system via simulation results. Finally conclusions are made in Section 7.

## 2 ADAPTIVE OFDM

The block diagram of adaptive OFDM system is depicted in figure 1. The binary bit stream is fed into the source encoder for encoding them. Then, the encoded serial bit stream is converted to parallel format which in further sent to adaptive modulator. The goal of adaptive modulator is to choose the appropriate modulation mode for transmission in each subband, given the local SNR, in order to achieve good trade-off between spectral efficiency and overall BER. Two types of modulation schemes have been used as the option for adaptation mechanism. Rest of the part in transmitter is just same as the traditional OFDM transmitter. The channel estimation and mode selections are done at the receiver side and the information is sent to the transmitter using a feedback channel [12]. In this model the adaptation is done frame by frame. The channel estimator is used to estimate the instantaneous SNR of the received signal. Based on the

instantaneous SNR calculated, the best mode will be chosen for the next transmission frame. This task is done by the mode selector block. At the transmitter the adaptive modulator block consists of different modulators block consists of different modulators which are used to provide different modulation modes. The switching between these modulators will depend on the instantaneous SNR.



In this paper, sub-band adaptive transmission schemes are employed to reduce the complexity. In sub-band adaptive OFDM transmission, all subcarriers in an AOFDM symbol are split into blocks of adjacent subcarriers referred to as subbands. The same mode is employed for all subcarriers of the same sub-band.[10]. The choice of the modes to be used by the transmitter for its next OFDM symbol is determined by the channel quality estimate of the receiver based on the current OFDM symbol. Perfect channel estimation is assumed in this paper. In this simulation the instantaneous SNR of the subcarrier is measured at the receiver. The channels quality varies across the different subcarriers for frequency selective channels. The received signal at any subcarrier can be expressed as:

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$$R_n = H_n X_n + W_n$$
(1)

Where,  $H_n$  is the channel coefficient at any subcarrier,  $X_n$  is the transmitted symbol and  $W_n$  is the Gaussian noise sample.

So the instantaneous SNR can be calculated using,  

$$SNR_n = \frac{H_n^2}{N_0}$$
(2)

Where,  $N_0$  is the noise variance [7]. The conservative approach in threshold based adaptation is by using the lowest quality subcarrier in each sub-band for controlling the adaptation algorithm [11]. It means that the lowest value of SNR will be used in mode selection. By using this method, the overall BER in one sub-band is normally lower than the BER target. If the overall BER can be closer to the BER target by choosing a more suitable modulation mode or code rate, the throughput of the system will be higher [5]. Therefore a better adaptation algorithm is used in this paper to provide a better tradeoff between throughput and overall BER by choosing a more suitable scheme for each sub-band. Instead of using the lowest SNR in each sub-band, the average value of the SNR of the subcarriers in the sub-band is going to be used.

#### **3** RAYLEIGH FADING

The Rayleigh distribution is by far the most used one to model the fading phenomenon due to its simplicity, straight forward derivation from geometrical assumptions and fairly good agreement with experimental data. This is the case with no dominant line-of-sight link between transmitter and receiver, and the received real and imaginary parts of the fields are independent zero-mean Gaussian random variables. The PDF is given by:

$$F_{P}(p) = \frac{1}{P_{f}} e^{\frac{-p}{P_{f}}}$$
(3)

Where,  $P_f$  again is the mean fast fading power, here equal to the mean power received. The received power under Rayleigh fading follows the above "exponential" PDF. The standard deviation of the received power in this case can be found to be Pf as well. The simple form of the exponential pdf results in a simple cdf:

$$F_{P}(p) = 1 - e^{\frac{-p}{P_{f}}}$$
(4)

In many problems, our interest lies in the probability that the received power will fall below a specified level. Recalling that the CDF represents P [Prec < p], the CDF is precisely the function to be used for such questions, with p the specified minimum power level in Watts. In the case of Rayleigh fading, if we have values of p that are small compared to the mean

received power Pf, the CDF can be simplified through a power series expansion of the exponential to:

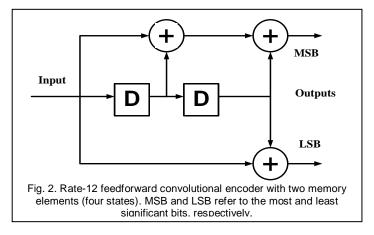
$$F_P(p) = 1 - (1 - \frac{p}{P_f}) = \frac{p}{P_f}$$
(5)

The power level that will exceeded q percent of the time is then,

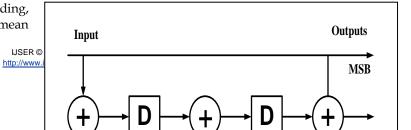
$$(1 - \frac{q}{100}) P_f$$
 (6)

#### **4** COVOLUTIONAL CODING

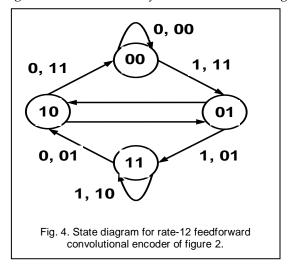
Convolutional codes represent one technique within the general class of channel codes. Channel codes (also called error-correction codes) permit reliable communication of an information sequence over a channel that adds noise, introduces bit errors, or otherwise distorts the transmitted signal. Elias [16, 17] introduced convolutional codes in 1955. These codes have found many applications, including deepspace communications and voice band modems. Convolutional codes continue to play a role in low-latency applications such as speech transmission and as constituent codes in Turbo codes. Two reference books on convolutional codes are those by Lin and Costello [18] and Johannesson and Zigangirov [19].



As any binary code, convolutional codes protect information by adding redundant bits. A rate-k/n convolutional encoder processes the input sequence of k-bit information symbols through one or more binary shift registers (possibly employing feedback). The convolutional encoder computes each n-bit symbol (n > k) of the output sequence from linear operations on the current input symbol and the contents of the shift register(s). Thus, a rate k/n convolutional encoder processes a k-bit input symbol and computes an n-bit output symbol with every shift register update. Figure 1 and figure 2 illustrate feed forward and feedback encoder implementations of a rate-1/2 code.



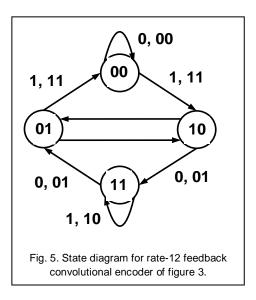
Convolutional encoders are finite-state machines. Hence, state diagrams provide considerable insight into the behavior of convolutional codes. Figures 4 and 05 provide the state diagrams for the encoders of Figures 1 and 2 respectively. The states are labeled so that the least significant bit is the one residing in the leftmost memory element of the shift register.



The branches are labeled with the 1-bit (single-bit) input and the 2-bit output separated by a comma. The most significant bit (MSB) of the two-bit output is the bit labeled MSB in figure 2 and figure 3. If one erases the state labels and the single-bit input labels, the remaining diagrams for figure 4 and figure 5 (labeled with only the 2-bit outputs) would be identical. This illustrates that the two encoders are equivalent in the sense that both encoders produce the same set of possible output sequences (or code words). Strictly speaking, a code refers to the list of possible output sequences without specifying the mapping of inputs sequences to output sequences. Thus, as in the above example, two equivalent encoders have the same set of possible output sequences, but may implement different mappings from input sequences to output sequences. In the standard convolutional coding application of transmission over additive white Gaussian noise (AWGN) with Viterbi decoding, such encoders give similar BER performance, but the different mappings of inputs to outputs do lead to small performance differences. The three-branch paths emphasized with thicker arrows in figure 4 and figure 5 are each the shortest nontrivial (i.e., excluding the all-zeros self-loop) loop from the all zeros state back to itself. Notice that for figure 4, the state diagram corresponding to the feed forward encoder, this loop requires only a single nonzero input. In contrast, for the state diagram corresponding to figure 5, this loop requires three nonzero inputs. In fact, for figure 5 no nontrivial loop from the all-zeros state to itself requires fewer than two nonzero inputs. Thus the feed forward shift register has a finite impulse response, and the feedback shift register has an infinite impulse response. This difference is not particularly important for convolutional codes decoded with Viterbi, but it is extremely important to convolutional encoders used as constituents in Turbo codes, which are constructed by

concatenating convolutional codes separated by interleavers. Only feedback encoders (with infinite impulse responses) are effective constituents in Turbo codes. Thus, equivalent encoders can produce dramatically different performance as constituents in Turbo codes, depending on whether or not they meet the requirement for an infinite impulse response.

Convolutional code decoding algorithms infer the values of the input information sequence from the stream of received distorted output symbols. There are three major families of decoding algorithms for convolutional codes: sequential, Viterbi, and maximum a posteriori (MAP). Wozencraft proposed sequential decoding in 1957 [20]. Fano in 1963 [21] and Zigangirov in 1966 [22] further developed sequential decoding. In 1974, Bahl et al. [23] proposed MAP decoding, which explicitly minimizes bit (rather than sequence) error rate. Compared with Viterbi, MAP provides a negligibly smaller bit error rate (and a negligibly larger sequence error rate). These small performance differences require roughly twice the complexity of Viterbi, making MAP unattractive for practical decoding of convolutional codes. However, MAP decoding is crucial to the decoding of Turbo codes. For the application of MAP decoding to Turbo codes, see the original paper on Turbo codes by Berrou et al. [24] and Benedetto et al.'s specific discussion of the basic turbo decoding module [25]. When convolutional codes are used in the traditional way (not as constituents in Turbo codes), they are almost always decoded using some form of the Viterbi algorithm, and the rest of this section focuses on describing it. The goal of the Viterbi algorithm is to find the transmitted sequence (or codeword) that is closest to the received sequence. As long as the distortion is not too severe, this will be the correct sequence.



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#### 5 PROPOSED AOFDM SYSTEM MODEL

The block diagram of the proposed AOFDM is depicted in figure 6. In the modified model, channel coding is introduced just after the source coder and before the serial to parallel converter. In a typical communication system the channel coding is performed before modulation and after source coding. The same fashion is maintained here. Convolutional coding is the chosen scheme for channel coding. So each parallel data from serial to parallel converter is convolutionally encoded which is further modulated in the process of OFDM mechanism. That means the inputs of the efficient OFDM system are convolutionally encoded which has an elevated possibility to achieve a better BER performance of the overall system.

Binarv Serial to Source Adaptive Convolution Paraller Data Encoder Encoder Modulato Stream Converter Mode Selector Parallel to Cyclic Serial Prefix IFFT ransmitter Converter Extension Ravleigh Fading Channel Cyclic Serial to onvolutiona Paraller Prefix FFI Receiver Decoder Removal Converter Channel Estimato Parallel to Binary Adaptive Source Serial emodulato Data Decoder Converter Stream TABLE 4 SWITCHING THRESHOLD FOR MPSK WITHOUT TRANSMISSION BLOCKING ER©2 www.ij Modulation Mode Thresholds Schemes 1  $SNR \le 11 \text{ dB}$ No Transmission

The switching threshold for activating different modes can be determined by extensive simulation of the fixed mode modulation system. The switching algorithm used for the adaptive modulation schemes are presented in Table 1 till Table 4. Two types of adaptation modes will be used. The first one is adaptive modulation without transmission blocking and the second one is adaptive modulation with transmission blocking. In adaptive modulation without transmission blocking, data will be constantly transmitted in this scheme even though the channel is in deep fades. If the channel quality is very bad, a robust modulation mode will be used and when the channel quality is good a spectrally efficient modulation will be used. In adaptive modulation with transmission blocking, transmission will be disabled when the channel is in deep fade. This mode is introduced because the signal quality is too bad to guarantee a required transmission [13]. Data will be transmitted if the channel quality improved.

TABLE 1				
Switching Threshold for Mqam Without Transmission Blocking				
Mode	Thresholds	Modulation		
		Schemes		
1	SNR ≤ 15.6 dB	4QAM		
2	15.6 dB < SNR ≤ 18.6 dB	8QAM		
3	$18.6 \text{ dB} \le \text{SNR} \le 21.5 \text{ dB}$	16QAM		
4	$21.5 \text{ dB} \le \text{SNR} \le 24.6 \text{ dB}$	32QAM		
5	SNR > 24.6 dB	64QAM		

TABLE 2				
Switching Threshold for Mqam With Transmission Blocking				
Mode	Thresholds	Modulation Schemes		
1	$SNR \le 10 \text{ dB}$	No Transmission		
2	10 dB < SNR ≤ 15.6 dB	4QAM		
3	15.6 dB < SNR ≤ 18.6 dB	8QAM		
4	18.6 dB < SNR ≤ 21.5 dB	16QAM		
5	21.5 dB< SNR ≤ 24.6 dB	32QAM		
	SNR > 24.6 dB	64QAM		

Mode	Thresholds	Modulation Schemes		
SWITCHING THRESHOLD FOR MPSK WITH TRANSMISSION BLOCKING				
	TABLE 3			

**QPSK** 

8PSK

 $SNR \le 16.6 \text{ dB}$ 

 $16.6 \text{ dB} \le \text{SNR} \le 22$ 

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2

## 6 SIMULATION AND ANALYSIS

In this section the employment of convolutional code in adaptive modulation system is investigated. It is known theoretically that, the usage of coding can improve the BER performance. The simulation parameters are illustrated below:

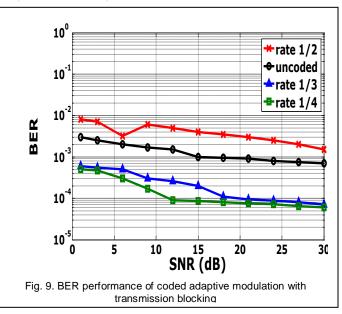
TABLE 5				
SIMULATION PARAMETERS				
Parameter	Value			
Number of subcarriers	512			
Number of sub-band	32			
Number of subcarriers per	16			
sub-band				
IFFT Size	512			
Modulation Scheme	MPSK, MQAM			
Carrier frequency	2GHz			
Guard Time Duration	128			
SNR	1 to 30 dB			
Coding rate	1/2, 1/3, 1/4			
Sampling frequency	5.4 MHz			
Frame size	6			
Bandwidth	5 MHz			
Channel Model	Rayleigh Fading Channel			

Figure 7 shows the comparative BER performance of coded and un-coded adaptive OFDM system without the employment of transmission block under Rayleigh Fading Condition. Here three BER performance curves for three coding rates of 1/2, 1/3 and 1/4 and another BER performance curve for un-coded version are demonstrated.

 $\mathbf{H} = \begin{bmatrix} \mathbf{10}^{0} & \mathbf{H} \\ \mathbf{10}^{-1} & \mathbf{H} \\ \mathbf{10}^{-1} & \mathbf{H} \\ \mathbf{$ 

It can be seen that the convolutional code provide a considerable improvement in BER performance for higher coding rates. For lower coding rate like 1/2, the BER performance in superior than uncoded AOFDM for SNR after 14 dB while uncoded version exhibits finer BER performance before 14 dB. However, for coding rate 1/3 and1/4, the BER performance is considerably superior. Coding rate 1/4 reveals the highest level of BER performance among all.

For adaptive modulation with transmission blocking, the comparable performance is observed. From figure 8, it is obvious that the BER performances progress tremendously. Specifically when rate 1/4 code is used, the BER performance decreases below 10-4 for SNR more than 6 dB. These results suggest that more reduction in the BER can be obtained by using channel coding.



From the analysis done, it can be seen that the BER target was achieved for all SNR when we utilized transmission blocking in comparison with an AOFDM convolutionally coded system without using transmission blocking under Rayleigh fading condition. Thus the advantage of using an adaptive convolutionally coded OFDM scheme with transmission blocking is that the performance can be designed to meet a certain required BER. However the disadvantage is that the

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utilization of the transmission blocking results in transmission latency.

#### 7 CONCLUSIONS

In this paper, the performances of adaptive transmission scheme for convolutionally coded OFDM under Rayleigh fading condition have been investigated. The advantage of employing convolutional coding in adaptive OFDM is revealed by comparing their performance with uncoded transmission system. A better adaptation algorithm is used to improve BER performance. This algorithm utilizes the average value of the instantaneous SNR of the subcarriers in the subband as the switching parameter. The results show an improved BER performance in a Rayleigh fading channel.

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